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LISA orbit selection and stability

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Abstract. The Laser Interferometer Space Antenna is a space mission designed to detect gravitational waves in the frequency range from below 0.0001 Hz to 1 Hz by measuring changes in the distance between spacecraft separated by several million kilometers. The spacecraft orbit in a triangular formation forming three (not independent) interferometers with arm lengths determined by the distances between the vertices. The nominal orbit configuration is described and contrasted with an alternative configuration. Changes in the distances between the vertices causes a Doppler shift in the laser signals between spacecraft. The size of the measurement error introduced by this Doppler shift is dependent on the stability of the spacecraft formation.

1. Introduction

The current design for the Laser Interferometer Space Antenna (LISA) mission is to have six spacecraft in a triangular formation with two spacecraft at each vertex and five million kilometers between vertices [1]. Changes in the distance between spacecraft are measured using laser interferometry in order to detect gravitational waves. The pair of spacecraft at each vertex perform as the center of a Michelson-like interferometer with arm length given by the distance to the spacecraft at the other vertices. The orbits for the individual spacecraft are chosen to maintain the formation throughout a two year nominal mission.

The frequency response of the interferometers to gravitational waves is dependent on the arm length. The detector response at higher frequencies is limited by reduced change in the separation between spacecraft (for a given gravitational wave amplitude) for signals with wavelengths less than the distance between spacecraft. The detector response at low frequencies is limited by thermally-induced noise and other disturbances of the test masses inside each spacecraft. For a given level of noise a greater distance between spacecraft will give improved sensitivity for low-frequency gravitational signals. By choosing the spacecraft orbits to keep the distance between distant spacecraft nearly constant, the frequency response of the mission will be kept at the chosen optimum. Keeping the distance between spacecraft as constant as possible also reduces problems caused by Doppler shifts of the lasers signals between spacecraft, as will be discussed below.

Two candidate orbit geometries were proposed in initial studies of the LISA mission. The heliocentric option is shown in figure 1 a [1][2]. In the heliocentric option the spacecraft form a triangle with a center a fixed distance behind the Earth; from the Earth the triangle appears to rotate about the center with a period of one year.

Each spat.c.c.rafL is in an orbit about the sun with major axis $D = 2AU$ and eccentricity $e = d/(D\sqrt{3})$, where d is the separation between vertices. If the spacecraft were all in the same plane then the separation between spacecraft would vary between De and $De/2$. By giving the spacecraft an inclination $i = d/D$, and by appropriate choice of the node, anomaly, and argument of perihelion, the separation between spacecraft is constant to order De^2. In the geocentric option, the spacecraft would be in a triangular formation ill the ecliptic plane centered 011 the Earth, as shown in figure 1b [3]. The formation has arm lengths of one million kilometers and rotates in the retrograde direction with a period of approximately 53 days.

Each of the orbit options has advantages. The heliocentric formation has the property that the directions between spacecraft are always within 30 degrees of being orthogonal to the direction to the sun. This allows the spacecraft to be designed such that sunlight never enters the interferometer optics, and also allows the spacecraft to have the sun always illuminating the! same part of the spacecraft. In contrast the geocentric formation rotation causes the direction between each pair of spat.mr'afl to coincide with the direction to the sun twice every 53 days. This means that a narrow-band filter is needed in the optical path of the geocentric spacecraft to reduce the optical and thermal noise associated with direct sunlight. The constantly changing aspect angle of the sun means that solar cells must be arranged on the geocentric spacecraft so that electrical energy can be produced over 360 degrees of rotation as opposed to the smaller solar array needed for the heliocentric spacecraft. The changing aspect angle of the sun 011 the geocentric spacecraft also causes thermal variations in the spacecraft that may affect the measurement, though to first order these should appear with a 53 day period that is not in the range in which gravitational waves are to be detected. For a given arm length the heliocentric formation has much smaller changes in the arm lengths, which simplifies the spacecraft design.

The geocentric option has the advantage that communications to the Earth requires smaller antennas and less transmitted power, due to the fact that the spacecraft are much closer to the Earth than for the heliocentric option, The geocentric option also requires less launch propellant for a given spacecraft mass. The heliocentric spacecraft need to be injected into a heliocentric transfer orbit and then require fairly large maneuvers to change the orbit inclinations and achieve the desired configuration [4]. The heliocentric transfer phase takes approximately thirteen months compared with under six months for the geocentric transfer phase.

In the studies carried out by ESA [5], the mission design for the geocentric orbit option included smaller optics, compared to the heliocentric mission design, in an attempt to reduce the mission cost. The reduced optics size led to a factor of five less sensitivity to gravitational waves. However, the initial studies showed only a small cost advantage for the geocentric mission option. Therefore for LISA the heliocentric mission option, with greater sensitivity, longer arm lengths (which allow for better instrumental sensitivity to gravitational waves at frequencies up to $3mHz$), and better thermal and dynamic stability, was selected as the baseline and has been studied in more detail.

2. Orbit stability requirements

For LISA, changes in the distance between a pair of spacecraft are measured by transmission of a continuous laser beam from one spacecraft to the second; the laser beam received by the second spacecraft is detected and re-transmitted to the first spacecraft. The first spacecraft measures the phase difference between the incoming and outgoing laser beams, with changes in this phase caused by changes in the distance between the spacecraft.

The spacecraft are drag-free with orbits determined by the gravitational forces on the test mass in the center of each spacecraft clue to the sun and other solar system bodies. The nominal distance between spacecraft is not constant; for the heliocentric case changes in the arm length arc caused by the eccentricity of the orbits and by perturbations from the Earth and the other planets, while for the geocentric orbits the variation in the gravitational pull from the sun and moon disturbs the nominally circular motion about the Earth. These orbital changes of distance between spacecraft will impose Doppler shifts on the interferometer signals that will have to be removed using on-board oscillators (clocks). Noise from the oscillators will then corrupt the distance measurements. The amount of noise introduced depends on the size of the Doppler shift and the performance of the oscillator,

Currently, the best spacecraft oscillators arc oven-stabilized crystals characterized by an Allan deviation of $\sim 10^{-13}$ for averaging times of 1 to 1000 seconds, covering the principle regime of interest for LISA. A constant Allan deviation is characteristic of so-called flicker-frequency noise. For this type of noise the power spectral density of the phase noise S_ϕ, introduced at signal frequency f by the oscillator used to remove a frequency f_D is related to the Allan deviation σ_A by [6]

$$S_\phi(f) = \sigma_A^2 f_D^2 f^{-3}/(2\ln 2) \qquad (radian^2/Hz)$$

The Doppler shift due to an arm rate-of-change v is $f_D = 2\nu(v/c)$ where c is the speed of light and ν is the laser frequency, since a Doppler shift occurs in both directions of travel for the round-trip distance measurement. The measurement noise introduced is

$$\sqrt{S_x(f)} = \lambda/(2\pi)\sqrt{S_\phi(f)} \qquad (m/\sqrt{Hz})$$

where $\lambda = c/\nu$ is the laser wavelength. As a numerical example, if the arm rate-of-change is $1m/s$ and the laser wavelength is $\lambda = 1\mu m$ then the Doppler shift is $2MHz$ and the measurement noise! introduced at an observation frequency of $1mHz$, given the assumed oscillator performance, is $\sqrt{S_x} \approx 850pm/\sqrt{Hz}$. The measurement goal is $40pm/\sqrt{Hz}$. Keeping the noise introduced by the oscillator to an acceptable level thus requires arm rates-of-change of much less than $1m/s$, a much improved spacecraft oscillator, or some other means of canceling this error source.

3. Orbit stability

The LISA spacecraft arc designed to be drag-free so that the only significant forces affecting the test masses at the center of each spacecraft, arc gravitational. In the simplest case the only free parameters that can be adjusted to minimize the arm rates-of change arc the initial positions and velocities of the test masses, which then move under the influence of the gravitational field of the sun and planets. For the heliocentric configuration the typical arm length changes due to the initial shape of the orbits are of order De^2 with a main period T of one year. For an arm length d = 5 x $10^6 km$, this implies a maximum arm rate-of-change of order $v = (2\pi/T)\, d^2/(3D) \approx 5m/s$. Perturbations due to the Earth and other planets cause larger changes in the arm lengths after a few years. The degradation is larger when the formation is nearer the

Earth. The current plan for LISA is to have the center of the triangular formation 20 degrees in ecliptic longitude behind the Earth. This distance was chosen as a compromise between the the desire to reduce the orbit perturbations due to the Earth and launch vehicle and telecommunications capabilities.

(For the geocentric option the solar perturbation on the Earth-centered circular orbits gives arm rates-of-change of order $50m/s$ [3]. Recall that the geocentric option was for arm lengths of $1 \times 10^6 km$; for the same arm length the heliocentric option would have arm rates-of-change of order $1m/s$).

When the initial positions and velocities for the six spacecraft are chosen to minimize the average rate-of-change of the three arm lengths over a twe-year period, the arm rates-of-change are found to vary between $\pm 6m/s$ [4]. Given the current performance of space-qualified oscillators, removing the Doppler shifts of the nominal orbits introduces more noise in the measurement than can be tolerated.

Another option studied was to include occasional maneuvers by the spacecraft to reduce the arm rates-of-change. The idea is that instead of allowing the test masses to move under gravitational forces only for the entire two-year nominal mission, maneuvers could be done at intervals to keep the arm rates-of change smaller over given intervals. The maneuvers occur at each spacecraft mainly perpendicular to the direction between the spacecraft. The maneuvers serve to make small adjustments in the orbit period and eccentricity such that the arm lengths remain more constant. This strategy is limited by tile low level of thrust available from the small ion thrusters planned for the spacecraft. The thrusters are currently planned to have a maximum thrust of order 100μ N sufficient to counteract the force on the exterior spacecraft clue to the solar luminosity. With these thrusters it takes a long time to execute even small maneuvers, perhaps 1 day to change the velocity by $10cm/s$ (given the mass of the current spacecraft design). The noise force on the test masses during the execution of these maneuvers is assumed to be large! enough to preclude accurate measurements during that time.

Analysis has been done to show that it is not possible to keep the rates-of-change of all three arms of the heliocentric formation to an acceptable level using the ion thrusters [4]. It does appear feasible to stabilize two of the three arms to an acceptable level with a practical number of small maneuvers. If one particular vertex is considered as the prime vertex, then the same spacecraft oscillator can be used to remove the Doppler shift of the two arms meeting at the prime vertex. (With the current plan of two spacecraft at each vertex, the two spat.ccraft, can use the same oscillator by transmitting a signal from one spacecraft to the other with the oscillator phase encoded on the signal.) Then it is the difference in the rate-of-change of the two arms that introduces noise into the gravitational wave measurement. An orbit solution has been found with maneuvers taking place once each month, of magnitude $10cm/s$ or less, such that the difference in rate-of- change of the two prime arms is kept to an root-mean-square level of $7mm/s$. Using this difference velocity, the noise introduced by the oscillator with Allan deviation 10^{-13} is less than the goal of $40pm/\sqrt{Hz}$ goal. The disadvantages of using maneuvers to stabilize a pair of arms is that it does not allow for using the information available from the third aria and it involves a "dead time" of about one day each month. By not using the third arm the detector is sensitive to only one of the two possible gravitational wave polarizations at any given time. (The rotaLion of the formation over the annual period will cause a given pair of arms to be sensitive to different polarizations at different times.)

Another alternative to reduce the noise causal try the Doppler shifts is to modulate the laser beams with a signal based on the spacecraft oscillators, similar to the scheme discussed by Hellings et al. [7]. In this scheme each arm is essentially used as a delay line to stabilize the oscillators; the returned oscillator signal is compared to the local oscillator signal and the difference used to measure fluctuations in the spacecraft oscillator. This scheme has been adopted as the nominal plan for the I, ISA mission.

Within this scheme it is still advantageous to have the arm rates-of-change small since this reduces the dynamic range of the signal needed for the oscillator signal, For example, with arm rates-of-change of $15m/s$ and Doppler shifts of $30MHz$, it suffices to use a $200MHz$ modulation derived from the spacecraft oscillators on the laser signal [8], The modulation can be imposed using an electro-optical modulator already planned in the spacecraft payload for other purposes. This is somewhat simpler than the two-laser scheme outlined in Hellings et al. which is needed to account for the larger dynamic range associated with the geocentric orbit option.

With this clock-noise reduction scheme there are a variety of possible choices of nominal orbits that give acceptable ranges of Doppler shift over the period of the mission. The nominal design selected is to use initial orbits that could, if necessary, be adjusted by small maneuvers each month to keep the rates-of-change of onc pair of arms nearly the same throughout the mission. However no maneuvers are planned if performance is nominal. Figure 2 is a plot of the arm rates of one nominal scenario. (The orbits will change slightly in character depending on the chosen launch date.) In figure 2 the rate-of-change of arm length for two of the arms is almost identical fOr the first six months Of the mission. The difference ill rate-of-change Of these two arms could be kept small through the use of the small maneuvers. The third arm rate-of-change varies between $\pm 15m/s$ for the first two years, which is larger than necessary if all three arms arc treated equally, but is well within the capability of the electro-optical modulator to perform the clock cancellation scheme.

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4. References

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